# MULTIPLICATIVE THINKING

A Professional Development Seminar

presented by

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## OVERVIEW:

- A new focus
- From additive to multiplicative thinking: key concepts and strategies
- Concepts for multiplication and division
- Mental strategies
- Extending multiplication and division

## A NEW FOCUS

One of the main aims of school mathematics is to create **mental objects** in the mind's eye of children which can be manipulated flexibly with understanding and confidence.

A prolonged reliance on inefficient strategies such as 'make-all-count-all' is both developmentally dangerous and professionally irresponsible.

Dianne Siemon, 2000

## Introducing operation ideas:

Before children come to school they usually know what it means to:

- get more (addition – join and combine);
- have something taken-away, to have less than (subtraction – take-away, missing addend, and difference); and
- share equally (division – partition).

However, making and counting equal groups is not a natural part of their everyday experience.

## Preparing for multiplication:

Establish the value of equal groups through:

- sharing collections; and
- exploring more efficient strategies for counting large collections.

Explore concepts through **action stories** that involve naturally occurring 'equal groups' (e.g., the number of wheels on 3 toy cars, the number of fingers in the room) and situations that arise in stories from Children's Literature, eg, *Counting on Frank, The Doorbell Rang, One is a Snail, Ten is a Crab …*

## Multiplicative Thinking:

Multiplicative thinking is characterised by:

- A capacity to work flexibly and efficiently with an **extended range of numbers** (e.g., larger whole numbers, decimals, common fractions, ratio, & percent);
- An ability to **recognise and solve a range of problems** involving multiplication and/or division including direct and indirect proportion; and
- The **means to communicate** this effectively in a variety of ways (e.g., words, diagrams, symbolic expressions, and written algorithms).

**In short, multiplicative thinking is indicated by a capacity to work flexibly with the concepts, representations, and strategies of multiplication (and division) as they occur in a wide range of contexts.**

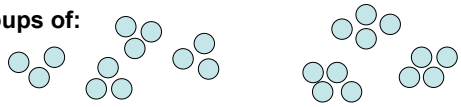
We also know a lot more about **how** children learn mathematics.

Meaningless rote-learning, mind-numbing, text-based drill and practice, and doing it one way, the teacher's way, does not work.

Concepts need to be **experienced**, strategies need to be **scaffolded** and EVERYTHING needs to be **discussed**.

## CONCEPTS FOR MULTIPLICATION:

**1. Groups of:**

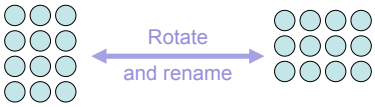


4 threes ... 3, 6, 9, 12 3 fours ... 4, 8, 12

Focus is on the group. Only suitable for small whole numbers, for example, it makes some sense to think about 6 eights, but very little sense in trying to think about 57 groups of 76 or 4.56 groups of 0.23

**Strategies**: make-all/count-all groups, repeated addition (or skip counting).

**2. Arrays:**




4 threes ... THINK: 6 and 6 3 fours ... THINK: 8, 12

Can see number in each group (equal groups), and the number of groups, but focus is on the product and representation supports commutativity (eg, 3 fours is the SAME AS 4 threes). This leads to more efficient mental strategies.

**Strategies**: mental strategies that build-on-from-known, eg, doubling and addition strategies

**NOTE: Arrays support a critical shift in thinking:**

**From** counting equal groups:
**1 three, 2 threes, 3 threes, 4 threes, ...**

That is, the traditional focus on the **number in each group** and **how many groups**

1 x **3**
2 x **3**
3 x **3**
4 x **3**
…

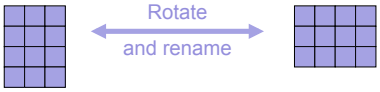
**To** a focus on the **number of groups**:
**3 ones, 3 twos, 3 threes, 3 fours, ...**

**3** x 1
**3** x 2
**3** x 3
**3** x 4
…

and **generalising**:
**3 groups of … is double the group and 1 more group.**

This introduces the **factor** idea for multiplication

**3. Regions:**




4 threes ... THINK: 6 and 6 3 fours ... THINK: 8, 12

Continuous model. Same advantages as *array* idea (discrete model) – Regions establish the basis for the subsequent 'area' idea and support more efficient mental strategies for multiplication.

**Supports:** partitioning and the construction of fraction diagrams, for example, 3 parts (thirds) by 4 parts (quarters) produces 12 parts (twelfths)

## CONSOLIDATING UNDERSTANDING:

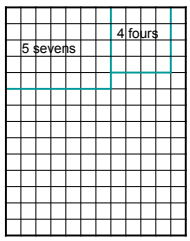
This can be achieved through games:

**For example, *MULTIPLICATION TOSS* ***

Each team/player needs a sheet of cm grid paper and 2 ten-sided dice (0 to 9).

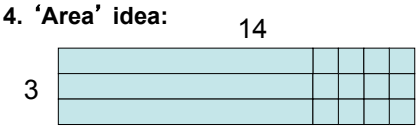
Players take it in turns to toss the dice. If a 5 and 7 are thrown, players can enclose either 5 rows of 7 (5 sevens) or 7 rows of 5. The game proceeds with no overlapping. The winner is the team/player with the most squares covered.

On any turn, a team/player can split their region into two separate regions, eg, 6 eights could be split into 4 eights and 2 eights or 3 eights and 3 eights to better fill in the spaces remaining.




*** Included in the Common Misunderstanding Material, DoE website**

**4. 'Area' idea:**




3 **by** 1 ten and 4 ones

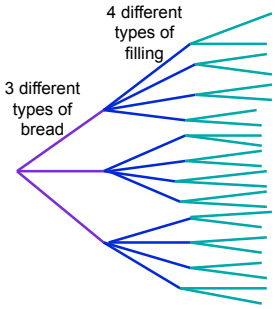
3 **by** 1 ten ... 3 tens

3 **by** 4 ones ... 12 ones

30 ... 42

Supports multiplication by place-value parts and the use of extended number fact knowledge, eg, 4 tens by 2 ones is 8 tens ... Ultimately, 2-digit by 2-digit numbers and beyond

**5. Cartesian Product or 'for each' idea:**



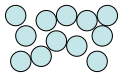

Eg, lunch choices

**3 x 4 x 2 = 24 different choices**

Supports 'for each' idea and multiplication by 1 or more factors

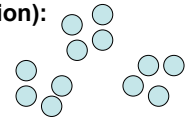
## CONCEPTS FOR DIVISION:

**1. How many groups in (quotition):**



12 counters

**How many fours in 12?**



1 four, 2 fours, 3 fours

Really only suitable for small collections of small whole numbers, eg, some sense in asking: How many fours in 24? But very little sense in asking: How many eights in 3074 or how many groups of 4.8 in 34.5?

**Strategies**: make-all/count-all groups, repeated addition

**Quotition (guzinta) Action Stories:**

24 tennis balls need to be packed into cans that hold 3 tennis balls each. How many cans will be needed?

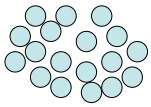
How many threes?

Sam has 48 marbles. He wants to give his friends 6 marbles each. How many friends will play marbles?

How many sixes?

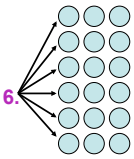
**Total** and **number in each group** known – Question relates to **how many groups.**

**2. Sharing (partition):**



18 counters

**18 sweets shared among 6. How many each?**



3 in each group

A more powerful notion of division which relates to the **array** and **region** models for multiplication and extends to fractions and algebra

**Strategy**: 'Think of Multiplication', eg, 6 what's are 18? 6 threes

**Partition Action Stories:**

42 tennis balls are shared equally among 7 friends. How many tennis balls each?

THINK: 7 what's are 42?

Sam has 36 marbles. He packs them into 9 bags. How many marbles in each bag?

THINK: 9 what's are 36?

**Total** and **number of groups** known – Question relates to **number in each group.**

## MENTAL STRATEGIES FOR MULTIPLICATION FACTS 0 x 0 TO 9 x 9:

- Doubles and doubles 'reversed' (twos facts*)
- Doubles and 1 more group ... (threes facts)
- Double, doubles ... (fours facts)
- Same as (ones and zero facts)
- Relate to ten (fives and nines facts)
- Rename number of groups (remaining facts)

* 2 ones, 2 twos, 2 threes, 2 fours, 2 fives, 2 sixes etc

## Traditional Multiplication 'Tables':

The 'traditional tables' are not really tables at all but lists of equations which **count groups**, for example:

| | |
|---|---|
| 1 x 3 = 3 | 1 x 4 = 4 |
| 2 x 3 = 6 | 2 x 4 = 8 |
| 3 x 3 = 9 | **3 x 4 = 12** |
| **4 x 3 = 12** | 4 x 4 = 16 |
| 5 x 3 = 15 | 5 x 4 = 20 |
| 6 x 3 = 18 | 6 x 4 = 24 |
| 7 x 3 = 21 | 7 x 4 = 28 |
| 8 x 3 = 24 | 8 x 4 = 32 |
| 9 x 3 = 27 | 9 x 4 = 36 |
| 10 x 3 = 30 | 10 x 4 = 40 |
| 11 x 3 = 33 | 11 x 4 = 44 |
| 12 x 3 = 36 | 12 x 4 = 48 |

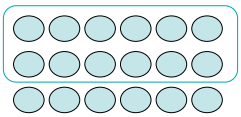
This is grossly inefficient

3 fours is not seen to be the same as 4 threes ...

10's and beyond not necessary

**Mental strategies build on experiences with arrays and regions:**

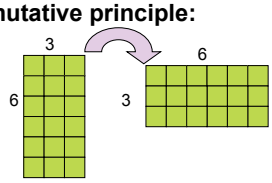
Eg, 3 sixes ... THINK:
double 6 ... 12, and 1 more 6 ... 18



3 rows of 6

**And the commutative principle:**

Eg, For 6 threes ...
THINK:
3 sixes ...
double 6, 12, and 1 more 6 ... 18




## A more appropriate multiplication 'table':

Uses a **region model** to support efficient, mental strategies based on the **factor** idea:

4 rows of 1

4 ones

| X | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 one | 1 two | 1 three | 1 four | 1 five |
| 2 | 2 ones | 2 twos | 2 threes | 2 fours | 2 fives |
| 3 | 3 ones | 3 twos | 3 threes | 3 fours | 3 fives |
| 4 | 4 ones | 4 twos | 4 threes | 4 fours | 4 fives |
| 5 | 5 ones | 5 twos | 5 threes | 5 fours | 5 fives |

## A more appropriate multiplication 'table':

Uses a **region model** to support efficient, mental strategies based on the **factor** idea:

4 rows of 2

4 twos

| X | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 one | 1 two | 1 three | 1 four | 1 five |
| 2 | 2 ones | 2 twos | 2 threes | 2 fours | 2 fives |
| 3 | 3 ones | 3 twos | 3 threes | 3 fours | 3 fives |
| 4 | 4 ones | 4 twos | 4 threes | 4 fours | 4 fives |
| 5 | 5 ones | 5 twos | 5 threes | 5 fours | 5 fives |

## A more appropriate multiplication 'table':

Uses a **region model** to support efficient, mental strategies based on the **factor** idea:

4 rows of 3

4 threes

| X | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 one | 1 two | 1 three | 1 four | 1 five |
| 2 | 2 ones | 2 twos | 2 threes | 2 fours | 2 fives |
| 3 | 3 ones | 3 twos | 3 threes | 3 fours | 3 fives |
| 4 | 4 ones | 4 twos | 4 threes | 4 fours | 4 fives |
| 5 | 5 ones | 5 twos | 5 threes | 5 fours | 5 fives |

## A more appropriate multiplication ‘table’:

Uses a **region model** to support efficient, mental strategies based on the **factor** idea:

Eg, 4s Facts:

Read across the row ⇨

| X | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 one | 1 two | 1 three | 1 four | 1 five |
| 2 | 2 ones | 2 twos | 2 threes | 2 fours | 2 fives |
| 3 | 3 ones | 3 twos | 3 threes | 3 fours | 3 fives |
| 4 | 4 ones | 4 twos | 4 threes | 4 fours | 4 fives |
| 5 | 5 ones | 5 twos | 5 threes | 5 fours | 5 fives |

“4 ones, 4 twos, 4 threes, 4 fours, … 4 of anything”

## This halves the learning as

**7 fours**

Can be rotated to show …

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 one 1 | 1 two 2 | 1 three 3 | 1 four 4 | 1 five 5 | 1 six 6 | 1 seven 7 | 1 eight 8 | 1 nine 9 |
| 2 | 2 ones 2 | 2 twos 4 | 2 threes 6 | 2 fours 8 | 2 fives 10 | 2 sixes 12 | 2 sevens 14 | 2 eights 16 | 2 nines 18 |
| 3 | 3 ones 3 | 3 twos 6 | 3 threes 9 | 3 fours 12 | 3 fives 15 | 3 sixes 18 | 3 sevens 21 | 3 eights 24 | 3 nines 27 |
| 4 | 4 ones 4 | 4 twos 8 | 4 threes 12 | 4 fours 16 | 4 fives 20 | 4 sixes 24 | 4 sevens 28 | 4 eights 32 | 4 nines 36 |
| 5 | 5 ones 5 | 5 twos 10 | 5 threes 15 | 5 fours 20 | 5 fives 25 | 5 sixes 30 | 5 sevens 35 | 5 eights 40 | 5 nines 45 |
| 6 | 6 ones 6 | 6 twos 12 | 6 threes 18 | 6 fours 24 | 6 fives 30 | 6 sixes 36 | 6 sevens 42 | 6 eights 48 | 6 nines 54 |
| 7 | 7 ones 7 | 7 twos 14 | 7 threes 21 | 7 fours 28 | 7 fives 35 | 7 sixes 42 | 7 sevens 49 | 7 eights 56 | 7 nines 63 |
| 8 | 8 ones 8 | 8 twos 16 | 8 threes 24 | 8 fours 32 | 8 fives 40 | 8 sixes 48 | 8 sevens 56 | 8 eights 64 | 8 nines 72 |
| 9 | 9 ones 9 | 9 twos 18 | 9 threes 27 | 9 fours 36 | 9 fives 45 | 9 sixes 54 | 9 sevens 63 | 9 eights 72 | 9 nines 81 |

… that it is the same as

**4 sevens**

**double, doubles**

**14 …14**

**28**

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 one 1 | 1 two 2 | 1 three 3 | 1 four 4 | 1 five 5 | 1 six 6 | 1 seven 7 | 1 eight 8 | 1 nine 9 |
| 2 | 2 ones 2 | 2 twos 4 | 2 threes 6 | 2 fours 8 | 2 fives 10 | 2 sixes 12 | 2 sevens 14 | 2 eights 16 | 2 nines 18 |
| 3 | 3 ones 3 | 3 twos 6 | 3 threes 9 | 3 fours 12 | 3 fives 15 | 3 sixes 18 | 3 sevens 21 | 3 eights 24 | 3 nines 27 |
| 4 | 4 ones 4 | 4 twos 8 | 4 threes 12 | 4 fours 16 | 4 fives 20 | 4 sixes 24 | 4 sevens 28 | 4 eights 32 | 4 nines 36 |
| 5 | 5 ones 5 | 5 twos 10 | 5 threes 15 | 5 fours 20 | 5 fives 25 | 5 sixes 30 | 5 sevens 35 | 5 eights 40 | 5 nines 45 |
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| 7 | 7 ones 7 | 7 twos 14 | 7 threes 21 | 7 fours 28 | 7 fives 35 | 7 sixes 42 | 7 sevens 49 | 7 eights 56 | 7 nines 63 |
| 8 | 8 ones 8 | 8 twos 16 | 8 threes 24 | 8 fours 32 | 8 fives 40 | 8 sixes 48 | 8 sevens 56 | 8 eights 64 | 8 nines 72 |
| 9 | 9 ones 9 | 9 twos 18 | 9 threes 27 | 9 fours 36 | 9 fives 45 | 9 sixes 54 | 9 sevens 63 | 9 eights 72 | 9 nines 81 |

## Doubles (twos):

2 ones, 2 twos, 2 threes, 2 fours, 2 fives ...

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

2 fours ... THINK: double 4 ... 8

⇩

2 sevens ... THINK: double 7 ... 14

⇩

But for 7 twos ... THINK: double 7 ... 14

## Doubles and 1 more group (threes):

3 ones, 3 twos, 3 threes, 3 fours, 3 fives ...

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

3 eights THINK: double 8 and 1 more 8 16 , 20, 24

⇩

But for 9 threes ... THINK?

⇩

3 twenty-threes?

## Doubles doubles (fours):

4 ones, 4 twos, 4 threes, 4 fours, 4 fives ...

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

4 sixes THINK: double 6 ... 12 double again, 24

⇩

But for 8 fours ... THINK?

⇩

4 forty-sevens?

## ‘Same as’ (ones and zeros):

1 one, 1 two, 1 three, 1 four, 1 five, ...

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

1 of anything is itself ... 8 ones, same as 1 eight

Cannot show zero facts on table ...
0 of anything is 0 ... 7 zeros, same as 0 sevens

## Relate to tens (fives and nines):

5 ones, 5 twos, 5 threes, 5 fours, 5 fives ...
9 ones, 9 twos, 9 threes, 9 fours, 9 fives ...

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

5 sevens THINK: half of 10 sevens, 35

⇩

8 fives ... THINK?

9 eights THINK: less than 10 eights, 1 eight less, 72

## Rename number of groups (remaining facts):

6 sixes, 6 sevens, 6 eights ... 7 sixes, 7 sevens, 7 eights ... 8 sixes, 8 sevens, 8 eights ...

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

6 sevens THINK: 3 sevens and 3 sevens, 42 ... OR 5 sevens and 1 more 7

8 sevens THINK: 7 sevens is 49, and 1 more 7, 56

## MENTAL STRATEGY FOR DIVISION:

- Think of multiplication

Eg, 56 divided by 7? …
THINK: 7 what’s are 56?
… 7 sevens are 49, 7 eights are 56
So, 56 divided by 7 is 8

Work with **fact families**:

What do you know if you know that 6 fours are 24? ⇨

4 sixes are 24,
24 divided by 4 is 6,
24 divided by 6 is 4,
1 quarter of 24 is 6,
1 sixth of 24 is 4

## INITIAL RECORDING:

Once strategies known, introduce initial recording to support place-value

$$\begin{array}{r} 6 \\ \underline{\times\ 4} \\ 24 \end{array}$$

Read as **4 sixes** … THINK: doubles, doubles …
ASK: What do we know about 24?
4 ones and 2 tens … record ones with ones, and the tens with tens

$$\begin{array}{r} 8 \\ \underline{\times\ 6} \\ 48 \end{array}$$

Read as **6 eights** … THINK: 5 eights and 1 more eight … 40, 48
ASK: What do we know about 48?
8 ones and 4 tens … record ones with ones and tens with tens

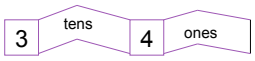
## Multiply 2-digit by 1-digit numbers:

Mentally:

THINK:
7 by 3 tens, 21 tens, and 7 fours
… 210 and 28 … 238 … OR? …

Eg, for 34 x 7

**Using Number Expanders:**




$$\begin{array}{r} \cancel{2}\ \ 4 \\ \times\ \ 7 \\ \hline 2\ 3\ 8 \end{array}$$

7 by 4 ones …28 ones

Record ones with ones and tens to regroup

7 by 3 tens … 21 tens, and 2 more tens, 23 tens

Record with the tens

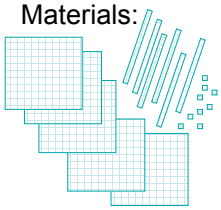
## Divide whole numbers by ones:

Mentally:

Eg, for 569 ÷ 8

THINK:
8 what's are about 569?
8 by 7 tens is 56 tens … 560
enough for 1 more eight … so
71 and 1 remainder

Materials:



Can we share hundreds among 8? No, trade for tens.

Can we share 56 tens among 8? Yes, 7 each

What's left to share? 9 ones, 1 each and 1 remaining

| 56 | tens | 9 | ones |
|---|---|---|---|

## CURRICULUM EXPECTATIONS:

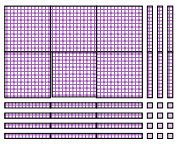
### By the end of Year 5 ...

- Identify and describe factors and multiples of whole numbers and use them to solve problems (ACMNA098)…
- Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies (ACMNA100)
- Solve problems involving division by a one-digit number including those that result in a remainder (ACMNA101)…
- Use equivalent number sentences involving multiplication and division to find unknown quantities (ACMNA121)

## Multiply 2-digit by 2-digit numbers:

33

24



Use MAB to support 'area' concept

```
    33
1  x 24
   132
   660
   792
```

**Ones by ones ...**
4 ones by 3 ones is 12 ones
Record 2 ones and 1 ten to regroup
**Ones by tens ...**
4 ones by 3 tens is 12 tens and 1 more ten, 13 tens, record
**Tens by ones ...**
2 tens by 3 ones is 6 tens
Record 6 tens and 0 ones
**Tens by tens ...**
2 tens by 3 tens is 6 hundreds
Record 6 hundreds
Add to find total

## Multiply decimals and fractions by ones:

| 3 | ones | 6 | tenths | 8 | hundredths |
|---|---|---|---|---|---|

```
2        3   x 4
1 4 . 7 2
```

Language?

4 by 8 hundredths .....

4 by 6 tenths ...

4 by 3 ones ....

Language?

```
 6¾
 x 4
2 7
```

4 by 3 quarters, 12 quarters

0 parts, 3 ones to regroup

4 by 6 ones, 24 ones and 3 more ones, 27

## Divide whole numbers and decimals by ones:

```
8 )458
```

Can I **share 4 hundreds** among 8? No.
Trade hundreds for tens
Can I **share 45 tens** among 8? Yes ...
How many left to share? 5 tens
Trade tens for ones

```
   5
8 )458
```

Can I **share 58 ones** among 8? Yes ...
How many left to share? 4 ones
Rename as tenths
Can I **share 20 tenths** among 8?Yes ...
How many left to share? 4 tenths
Rename as hundredths

```
   57.25
8 )458.00
```

Can I **share 40 hundredths**? Yes ...
How many left to share? None

## CURRICULUM EXPECTATIONS:

### By the end of Year 7 ...

•Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)…

•Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)…

•Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)…

•Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)

•Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)

•Find percentages of quantities and express one quantity as a percentage of another …(ACMNA158)

•Recognise and solve problems involving simple ratios (ACMNA173

•Investigate and calculate 'best buys'… (ACMNA174)…

**With and without the use of digital technologies**